

# Final Report on DSN Telemetry System Performance With Convolutionally Coded Data: Maximum Likelihood Decoding

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*This report finalizes the analysis of DSN telemetry system performance based on the convolutionally coded data for the short constraint length  $7\frac{1}{2}$  codes at low-bit rates, 8 to 2048 bits per second, obtained from CTA 21 for the block III type, S-band configuration. The results indicate that a loss of one or more decibels in the system performance may be expected due to system degradation. Also, burst error lengths up to 100 bits may not be unusual in actual operational situations.*

## I. Introduction

This is a final report on DSN telemetry system performance with the convolutionally coded data obtained from the Compatibility Test Area (CTA 21) during the 1975 calendar year (Ref. 1). The report focuses on the test results analyzed for the short constraint length  $7\frac{1}{2}$  codes to be employed by outer-planet Mariner spacecraft such as the Mariner Jupiter-Saturn (MJS). Analysis of the telemetry system for the long constraint length  $32\frac{1}{2}$  codes is presented separately elsewhere in this issue of the DSN Progress Report (Ref. 2). The tests cover bit rates ranging from 8 to 2048 bits per second (b/s) and modulation indexes (MI) from 37.2 to 75 deg.

To obtain the performance characteristics, a stream of fixed, known symbol pattern that can be decoded by both the Pioneer  $32\frac{1}{2}$  and MJS  $7\frac{1}{2}$  decoders, generated by the

Simulation Conversion Assembly (SCA), was modulated on the subcarrier and passed to the test transmitter, where the carrier was modulated (See Fig. 1a). The microwave equipment (UWV) routed the transmitted carrier to the receiver (RCVR), where a Y-factor was used to calibrate the input signal-to-noise ratio (Ref. 3). The symbol stream was demodulated in the Subcarrier Demodulator Assembly (SDA) and sent to the Symbol Synchronizer Assembly (SSA) for synchronization and quantization. The eight-bit A-D conversion of each symbol was recorded on magnetic tape by the Telemetry & Command Processor (TCP) in octal. On the UNIVAC 1108, the octal symbols were converted to FIELDATA characters before they were preprocessed to three-bit symbols (See Fig. 1b). The three-bit symbols were then processed and analyzed by the Viterbi Decoding Program (Ref. 4) to provide the final results for this report.

## II. Methodology and Analysis

DSN telemetry system performance may be described in terms of bit signal-to-noise ratio ( $E_b/N_0$ ), bit error rate (BER), or bit error probability (BEP), and bit error distribution. To obtain such parameters, the corrupted eight-bit symbols recorded on magnetic tape are converted to three-bit symbols, which are acceptable by the Viterbi Decoding Program that was designed to simulate as closely as possible the actual JPL Maximum Likelihood Convolutional Decoder (MCD). The program decodes the symbols and compares the result with the original data bits.

To reduce the amount of the decoded bits to a size that can be handled, the decoded bits and the original data bits are exclusive-ored and packed into a sequence of integers:<sup>1</sup>

$$g = (g_1, g_2, \dots, g_n)$$

where each  $g_i$  is the count of good bits (it will be referred to here as an error-free run, EFR), and  $n$  is the total number of EFRs for the test. In this notation, two consecutive errors occur when  $g_i$  equals zero. For example, the sequence (2,5,1,7,0,1,3) represents the event (0010000010100000001101000), where each 1 designates an error and each 0 designates a good bit. Since the decoded data stream ends with or without an error, this end bit will not be considered.

Now, if  $E$  is the total number of errors, then

$$E = n - 1$$

Representing the total number of good bits by  $G$ , we have

$$G = \sum_{i=1}^n g_i$$

Hence, the total number of decoded bits,  $D$ , for the test is

$$D = G + E$$

Next, if we denote an integer guardspace by  $g_s$ , a burst of bit errors is defined as a subsequence of  $g$  whose elements assume values less than  $g_s$ . That is, the EFR sequence of a burst  $b$  is of the form:

$$g_i, b, g_{i+k}$$

<sup>1</sup>An alternate way which was employed in Ref. 2 is to produce a sequence of integers  $(k_1, k_2, \dots, k_n)$ , where  $k_i$  is the number of consecutive good bits if  $i$  is even, otherwise  $k_i$  is the number of consecutive bad bits.

where

$$g_i, g_{i+k} \geq g_s$$

and

$$b = (g_{i+1}, g_{i+2}, \dots, g_{i+k-1})$$

$$g_j < g_s \text{ for } i+1 \leq j \leq i+k-1$$

The total number of errors in  $b$ ,  $\epsilon$ , is then

$$\epsilon = (i+k) - i = k$$

The total number of good bits contained in  $b$ ,  $\delta$ , is

$$\delta = \sum_{j=1}^{k-1} g_{i+j}$$

and the total number of bits in  $b$ , burst length, is

$$\epsilon + \delta$$

In general,  $g$  is of the form:

$$g = (g_1, b_1, g_2, b_2, \dots, b_m, g_m)$$

where  $g_i \geq g_s$  and each  $b_j$  contains  $g_k < g_s$ . The average burst length of the sequence  $g$  is

$$b_{ave} = \frac{b_1 + b_2 + \dots + b_m}{m}$$

and the average number of errors within a burst is

$$\epsilon_{ave} = \frac{\epsilon_1 + \epsilon_2 + \dots + \epsilon_m}{m}$$

where  $\epsilon_i$  is the number of errors within the  $i$ th burst. Note that the burst definition given above also includes isolated bit errors. However, the results described in Section IV do not consider isolated errors as bursts.

Now for an EFR  $r_i$ , let  $f_i$  denote its frequency of occurrence,  $c_i$  denote its cumulative number of  $f_i$ , and  $p_i$  denote its cumulative probability of  $c_i$ . Then,

$$(r_1, r_2, \dots, r_{k-1}, r_k)$$

$$(f_1, f_2, \dots, f_{k-1}, f_k)$$

$$(c_1, c_2, \dots, c_{k-1}, c_k)$$

$$(p_1, p_2, \dots, p_{k-1}, p_k)$$

are defined by

$$c_k = f_k, c_{k-1} = c_k + f_{k-1}, c_i = c_{i+1} + f_i, \dots, c_1 = c_2 + f_1$$

and

$$p_k = c_k/c_1, p_{k-1} = c_{k-1}/c_1, p_i = c_i/c_1, \dots, p_1 = c_1/c_1$$

where  $r_j$  are sorted as  $r_1 < r_2 < \dots < r_k$  and  $k$  is the total number of distinct EFRs. The cumulative probability  $p_i$  is the probability that an EFR  $r_i$  is being exceeded; it is a sample distribution function of EFRs or, for short, an EFR sample distribution.

### III. Viterbi Decoding Performance of Memory Path Lengths 36 and 64

In the early phase of data processing, memory path length (MPL) was set to 36. However, it was later found that the JPL MCD uses MPL 64. To make certain that the results already processed are compatible and comparable to subsequent results, an investigation was conducted to determine the discrepancy between the performances of the two values. The investigation reveals that the difference in performance between these two values appears to remain within 0.2 dB and is much less at high-bit SNRs. This finding also assures that the results obtained from Deep Space Station (DSS) 62 (Ref. 2) are valid for use in the MJS flight project design. Note that the decoder used at the DSS 62 is an off-the-shelf LV7015 from LINKABIT, which employs MPL 36.

The performance of the Viterbi decoding algorithm for MPLs 36 and 64 was investigated via the Viterbi Decoding Program (Ref. 4), using data generated from another 1108 program. The results are tabulated in Tables 1 and 2 for values of  $E_b/N_0 = 2, 3, 4$ , and 4.3 dB. Figure 2 provides the plots,  $E_b/N_0$  vs BEP, of the data. For comparison, the figure also includes the baseband performance curves of the JPL functional requirements and of the prototype MCD acceptance test data at 250 kb/s. The figure shows that the Viterbi decoding performance of MPL 64 is everywhere better than that of MPL 36. This is especially true for low SNRs. The improvement in the performance diminishes towards high SNRs. Nonetheless, the overall improvement appears to be less than 0.2 dB. Observe that the majority selection performance improves significantly with MPL.

Figure 3 provides the curve of EFR size  $R$  vs probability that  $R$  is exceeded for  $E_b/N_0 = 3$  dB. The curve reveals that large-sized EFRs have a higher probability to occur for MPL 64 than for MPL 36.

## IV. Test Results and Discussions

The complete test plan for the project was provided in Ref. 1. It consisted of 6 series of tests, A through F, with a total of 33 tests. Series A and B tests had 6 tests each, 1 to 7, except 5. There were 5 tests in series C, E, and F, and 6 tests in series D. The data rates of the tests ranged from 8 to 2048 b/s and modulation indexes from 37.2 to 75 deg. And only Block III RCVR/SDA type was used in the test setup.

Again, the Y-factor method is employed to calibrate  $E_b/N_0$  to the receiver. Since this type of calibration usually lacks precision, the data output at the TCP are not recorded until the system is adjusted so that the SSA symbol error rate (SER) printouts are averaged to the theoretical SER, obtained from a Telemetry Analysis Program. Consequently, DSN telemetry system performance is described here in terms of the  $E_b/N_0$  derived from the output SSA SER vs BEP of the decoded bits, and the EFR sample distribution.

Since the SSA SER is only an estimate of the input SER due to noise in transmission, the derived  $E_b/N_0$  is also an estimate of the input  $E_b/N_0$  to the receiver.

The test results are now given. The plots of  $E_b/N_0$  vs BEP for test series A through F are depicted in Fig. 4. Again, the JPL functional requirements for the MCD and the prototype MCD acceptance test data at 250 kb/s are included in the plots. The plots also provide the corresponding system performance curves for majority selection. Figure 5 shows the EFR sample distribution plots of the above tests. The test descriptions and burst error statistics, including burst rate, average number of errors per burst, average burst length, average EFR length, and maximum burst error and burst length, are tabulated in Table 3. A burst rate is defined as the ratio of the number of bursts to the number of decoded bits.

In what follows, we shall, for simplicity, call each test and the data file of each test by only the test name. For example, test A and the data file of test A will be referred to as A.

The results of A1, A2, A3, and A6 satisfied the functional requirements. The first part of A1, up to bit 780012, was not included in the plot due to large burst errors; some burst lengths were as large as 88, which resulted in as high a BEP as  $5.75 \times 10^{-4}$ . After decoding up to 587608 bits, a burst error of length 50 occurred in A4. This was the only burst error found in the data file. Test A7 contained 3 EFRs, which ranged from  $4.8 \times 10^5$  to  $7.3 \times 10^5$  bits.

The results of B1, B2, and B6 remained pretty much within the functional requirements. The early portion of B2 effected a rather low BEP, as low as  $5 \times 10^{-6}$ . All in all, B2 was a good file. For B3, a burst of length 36 occurred after bit 1222432; this made the BER of B3 high. Large burst errors and EFRs tended to occur more towards the end of B7, otherwise, decoding of B7 proceeded normally.

Over one million bits were decoded for C1 and C4, and their BEPs remained within the functional requirements. No errors were found in C2, within 1.8 million bits. But two errors occurred in C3, immediately after bit 106768. Since  $E_b/N_o$  of C3 was above 6 dB, at least  $10^7$  bits were required to provide a meaningful result; however, C3 had only 1.4 million bits. The same may be said about C5. Note that due to large burst errors, the first and last portions of C5 were not included in the BEP calculation; the former bursts were as large as 34, and the latter bursts were even worse, as large as 100. The graph of the EFR sample distribution of the series C tests is shown in Fig. 5c. Observe that the curves elevate with  $E_b/N_o$ .

The series D test results deviated, towards the worse side, uniformly from the functional requirements, as much as one or more decibels. Nevertheless, the decoding of these tests proceeded normally.

Only E1 and E5 were obtained for the series E tests. It was not possible to get the receiver for E2 in lock, and, consequently, no data were recorded for this test. Similar to the series D tests, the results of E1 and E5 were noticeably worse than the functional requirements. Note that E3 contained no decoding errors, and E4 had an unrecognizable bit pattern.

For the series F tests, only F1, F2, and F3 were run. The results of these tests were also worse than the func-

tional requirements. Due to large burst errors, F1-2 was terminated before an end of file was reached. The decoder did not, however, encounter such a difficulty in F1-1. Similarly, no major difficulty was faced in F2. Also, no decoding errors were detected in the entire file of F3.

Finally, the following observations may be made about the EFR sample distribution curves in general:

- (1) The depth of each curve indicates the quantity of EFR, the deeper the greater.
- (2) The extent (to the right) of each curve indicates the size of its EFRs, the farther the larger.
- (3) Each transition signifies an EFR.

## V. Conclusions

The task undertaken was complicated by many intermediary steps in setting up the test equipment in the Compatibility Test Area and in processing the test data. However, the final test results appear to reveal that the performance of the DSN telemetry system seems to satisfy the system functional requirements, at least at the bit rates considered which range from 8 to 2048 b/s and under the various test conditions prescribed in this report. System performance degradation of one or more decibels in actual operations under certain conditions may be expected, especially at low-bit rates. Also, though the average burst lengths are generally less than 10, within our range of analysis, error burst lengths up to 100 may not be unexpected. Lastly, MCD performance improves with memory path length. But an improvement of not more than 0.2 dB may be expected when changing from memory path length 36 to 64. Also, MCD performance of best metric selection is almost always better than that of majority selection.

## Acknowledgment

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## References

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Table 1. Data used for comparison between memory path lengths 36 and 64

MPL	$E_b/N_0$ , dB	Total			Bit errors		Symbol errors		Bit error rate		Best metric			Largest burst length		
		Decoded bits	Bursts	EFR	Best metric	Majority	Symbol errors	Best metric	Majority	SER, %	Burst rate	Average, bits				
												Errors/ burst	Burst length			
															EFR	length
64	2.0	35937	35	272	271	293	7518	$7.54 \times 10^{-3}$	$8.15 \times 10^{-3}$	10.46	$9.74 \times 10^{-4}$	7.43	12.21	123.38		
36	2.0	34272	53	390	389	505	7152	$1.14 \times 10^{-2}$	$1.47 \times 10^{-2}$	10.43	$1.55 \times 10^{-3}$	7.23	11.57	86.88	37	58
64	3.0	215937	29	195	194	204	34191	$8.98 \times 10^{-4}$	$9.45 \times 10^{-4}$	7.92	$1.34 \times 10^{-4}$	6.66	10.83	1106.37	19	26
36	3.0	215965	40	229	228	387	34197	$1.06 \times 10^{-3}$	$1.79 \times 10^{-3}$	7.92	$1.85 \times 10^{-4}$	5.60	9.40	942.08	19	26
64	4.0	215937	1	6	5	5	24571	$2.32 \times 10^{-5}$	$2.32 \times 10^{-5}$	5.69	$4.63 \times 10^{-6}$	5.00	7.00	35988.67	5	7
36	4.0	215965	1	6	5	15	24575	$2.32 \times 10^{-5}$	$6.95 \times 10^{-5}$	5.69	$4.63 \times 10^{-6}$	5.00	7.00	35993.33	5	7
64	4.3	539937	0	1	—	—	54548	—	—	5.05	—	—	—	539937	—	—
36	4.3	539965	0	1	—	5	54550	—	$9.26 \times 10^{-6}$	5.05	—	—	—	539965	—	—

Table 2. JPL prototype MCD acceptance test data at 250 kb/s<sup>a</sup>

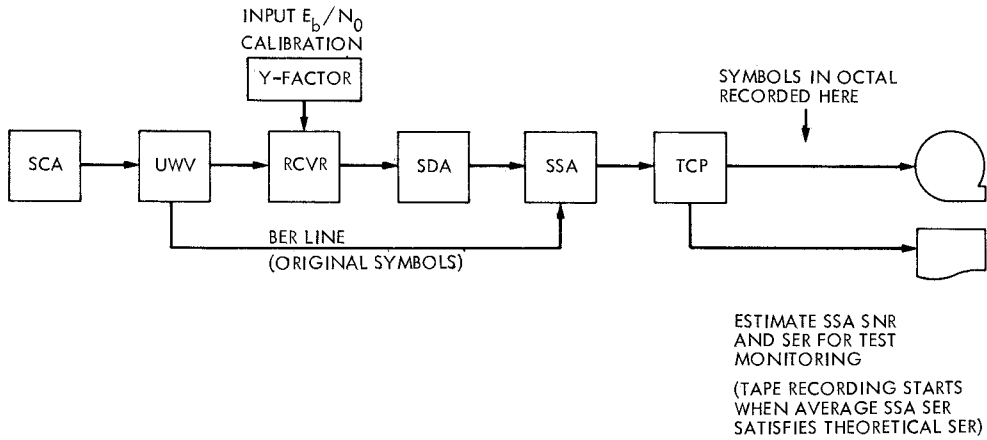
MPL	$E_b/N_0$ , dB	Total bits	Bit error rate	
			Total bit errors	Test data requirements
36	2.0	—	—	$1.00 \times 10^{-2}$
64	3.0	$4.10 \times 10^8$	3039	$7.41 \times 10^{-4}$
64	4.0	$4.10 \times 10^8$	1621	$3.96 \times 10^{-5}$
64	5.0	$6.55 \times 10^8$	89	$1.36 \times 10^{-6}$

<sup>a</sup>From M. Alberda, DSN Data Systems Development Section.

Table 3. Results for test series A through F

Test ID	MPL	$E_b/N_0$ , dB	No. of Decoded bits	Bit errors		Bit error rate			Best metric										
				Best Metric	Majority	MI, deg	SER, %	No. of Bursts	No. of EFR	Burst rate	Largest errors/burst length	Average, bits							
												Errors/burst	Burst length	EFR length					
A1	36	3.75	500256	69	147	55	55	$1.38 \times 10^{-4}$	$2.94 \times 10^{-4}$	6.18	17	70	$3.40 \times 10^{-5}$	8	14	3.88	6.71	9254	
A2	36	4.54	800000	4	7	55	55	$6.95 \times 10^{-5}$	$1.25 \times 10^{-4}$	4.58	1	5	$1.25 \times 10^{-6}$	4	8	4.00	8.00	246831	
A3	36	4.75	933120	5	5	55	55	$5.36 \times 10^{-6}$	$5.36 \times 10^{-6}$	4.21	1	6	$1.07 \times 10^{-6}$	5	7	5.00	7.00	66538	
A4	64	5.42	2877630	26	26	55	55	$9.04 \times 10^{-6}$	$9.04 \times 10^{-6}$	3.11	1	27	$3.48 \times 10^{-7}$	26	50	26.00	50.00	22601	
A6	64	4.61	1000000	7	7	67.6	75	$7.00 \times 10^{-6}$	$7.00 \times 10^{-6}$	4.46	2	8	$2.00 \times 10^{-6}$	4	6	3.50	5.50	124999	
A7	64	5.48	1800000	13	13	75	75	$3.72 \times 10^{-6}$	$3.72 \times 10^{-6}$	3.02	3	14	$1.67 \times 10^{-6}$	6	10	4.333	6.33	128570	
B1	36	3.41	600000	140	287	55	55	$2.33 \times 10^{-4}$	$4.78 \times 10^{-4}$	6.93	29	141	$4.83 \times 10^{-5}$	21	28	4.79	7.29	4477	
B2	36	3.93	2000000	65	65	55	55	$3.25 \times 10^{-5}$	$3.25 \times 10^{-5}$	5.81	16	66	$8.00 \times 10^{-6}$	6	14	4.00	6.88	—	
B3	64	4.94	2463901	109	116	55	55	$4.42 \times 10^{-5}$	$4.71 \times 10^{-5}$	3.88	16	110	$6.49 \times 10^{-6}$	22	36	6.75	10.88	22398	
B4	64	5.40	3736025	0	0	55	55	0.00	0.00	3.14	0	1	0.00	—	—	—	—	—	
B6	36	5.19	3946180	6	6	67.6	67.6	$1.52 \times 10^{-6}$	$1.52 \times 10^{-6}$	3.46	2	8	$5.06 \times 10^{-6}$	4	6	3.00	4.00	49370	
B7	36	5.38	1600000	65	65	75	75	$4.06 \times 10^{-5}$	$4.06 \times 10^{-5}$	3.17	10	52	$6.25 \times 10^{-6}$	11	21	5.30	8.40	28424	
C1	36	4.56	1010880	11	12	55	55	$1.08 \times 10^{-5}$	$1.19 \times 10^{-5}$	4.56	2	12	$1.98 \times 10^{-6}$	6	10	4.50	7.50	64799	
C2	64	5.25	1861801	0	0	55	55	0.00	0.00	3.37	0	1	0.00	0	0	0.00	0.00	1861801	
C3	64	6.32	1384209	2	2	55	55	$1.45 \times 10^{-6}$	$1.45 \times 10^{-6}$	1.93	1	3	$1.45 \times 10^{-6}$	2	2	2.00	2.00	461402	
C4	36	3.96	1010880	55	88	42	42	$5.44 \times 10^{-5}$	$8.71 \times 10^{-5}$	5.74	11	56	$9.89 \times 10^{-6}$	8	16	4.91	8.64	18050	
C5	64	5.32	1580038	5	5	67.6	67.6	$3.00 \times 10^{-6}$	$3.00 \times 10^{-6}$	3.26	2	6	$1.20 \times 10^{-6}$	2	4	2.00	3.00	283100	
D1	36	5.04	1123885	21	23	55	55	$1.87 \times 10^{-5}$	$2.05 \times 10^{-5}$	3.70	4	22	$3.56 \times 10^{-6}$	12	28	4.75	9.50	51084	
D2	64	5.49	1199457	8	8	55	55	$6.67 \times 10^{-6}$	$6.67 \times 10^{-6}$	2.98	3	9	$2.50 \times 10^{-6}$	3	5	2.33	3.33	133272	
D3	64	6.63	1000000	1	1	55	55	$1.00 \times 10^{-6}$	$1.00 \times 10^{-6}$	1.61	0	0	0.00	0	0	0.00	0.00	499999	
D5	64	6.20	1200096	1	1	58.4	66.5	$8.33 \times 10^{-7}$	$8.33 \times 10^{-7}$	2.06	0	0	0.00	0	0	0.00	0.00	600048	
D6	64	5.83	999873	444	444	66.5	66.5	$4.44 \times 10^{-4}$	$4.44 \times 10^{-4}$	2.52	85	445	$8.50 \times 10^{-5}$	7	14	3.29	5.29	2245	
E1	36	3.50	233280	89	187	42	42	$3.82 \times 10^{-4}$	$8.02 \times 10^{-4}$	6.74	18	78	4.50	9	20	4.50	8.17	2830	
E2								Not available											
E3	36	5.71	315468	0	0	42	42	0.00	0.00	2.69	0	0	0.00	0	0	0.00	0.00	315468	
E4								Unrecognized pattern											
E5	64	4.56	330561	27	27	45	45	$6.24 \times 10^{-5}$	$6.24 \times 10^{-5}$	4.56	6	28	$1.82 \times 10^{-5}$	4	6	3.50	4.67	11804	
F1-1	64	4.19	201465	68	68	42	42	$3.38 \times 10^{-4}$	$3.38 \times 10^{-4}$	5.26	15	69	$7.50 \times 10^{-5}$	7	15	4.47	6.87	2918	
F1-2	64	3.50	250000	89	92	42	42	$3.56 \times 10^{-4}$	$3.68 \times 10^{-4}$	6.74	16	78	$6.40 \times 10^{-5}$	11	23	4.81	8.69	2830	
F2	64	4.34	160209	14	14	42	42	$8.00 \times 10^{-5}$	$8.00 \times 10^{-5}$	4.98	4	15	$2.50 \times 10^{-5}$	5	7	3.50	3.75	10679	
F3	64	4.92	116865	0	0	42	42	0.00	0.00	3.90	0	1	0.00	0	0	0.00	0.00	116865	

(a) TELEMETRY TEST SETUP CONFIGURATION (IN CTA 21)



(b) TELEMETRY TEST DATA PROCESSING CONFIGURATION (UNIVAC 1108)

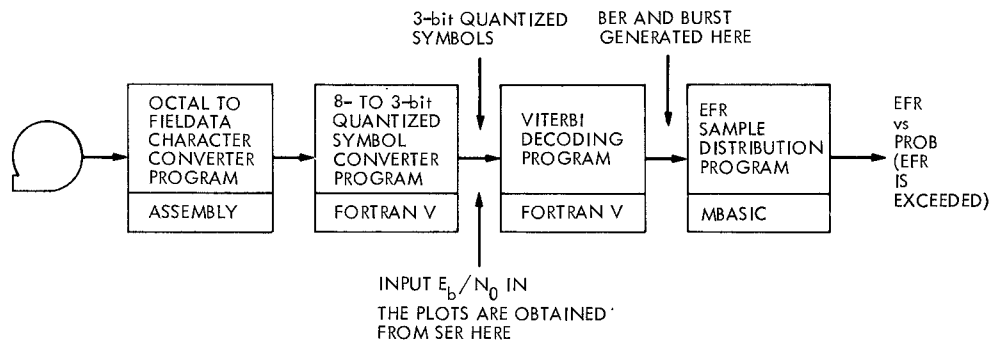


Fig. 1. Telemetry test setup and data processing configurations



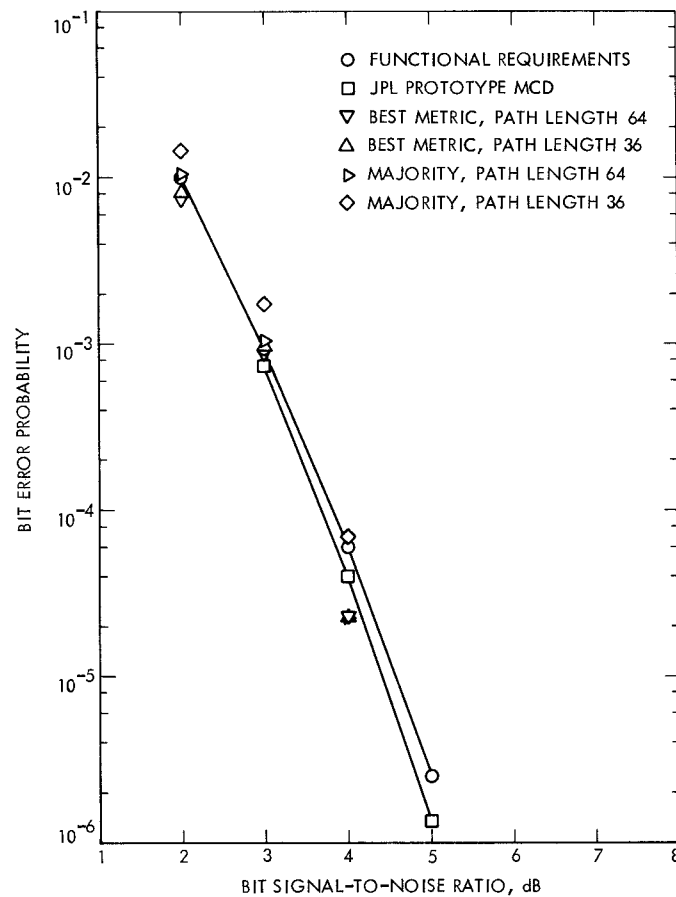


Fig. 2. Bit signal-to-noise ratio vs bit error probability

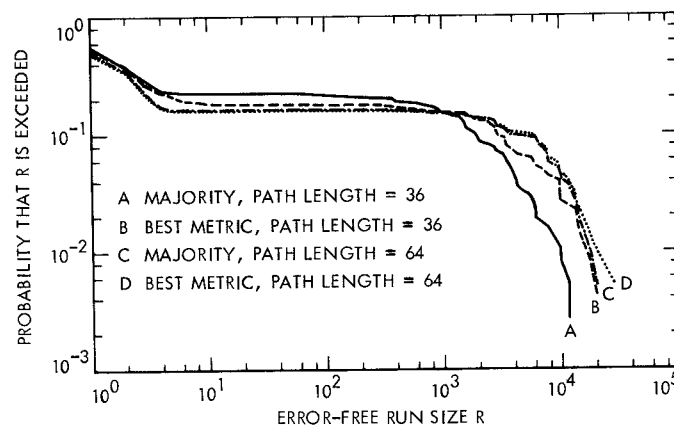


Fig. 3. Error-free run size  $R$  vs probability that  $R$  is exceeded

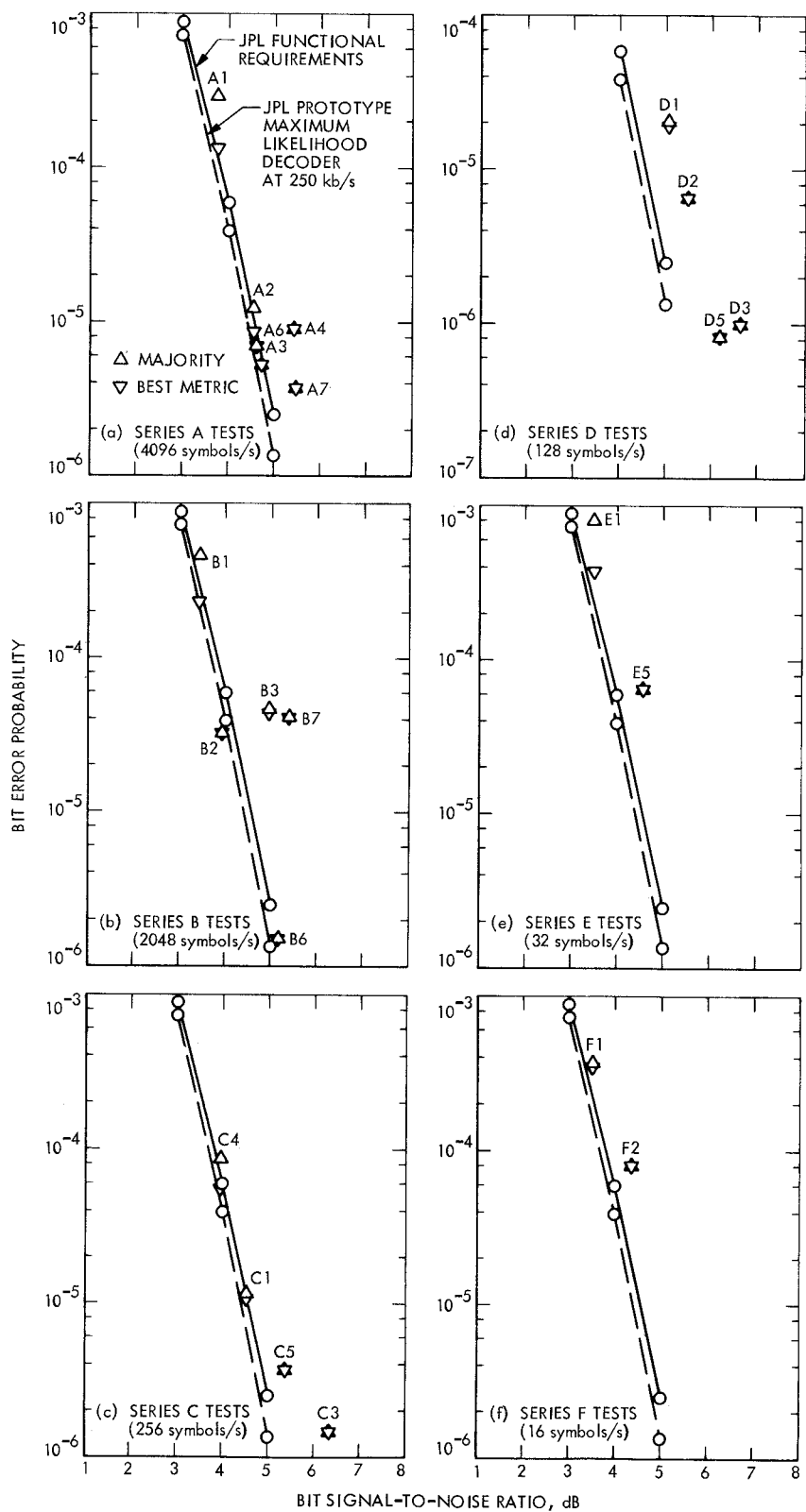


Fig. 4. Bit signal-to-noise ratio vs bit error probability for test series A through F

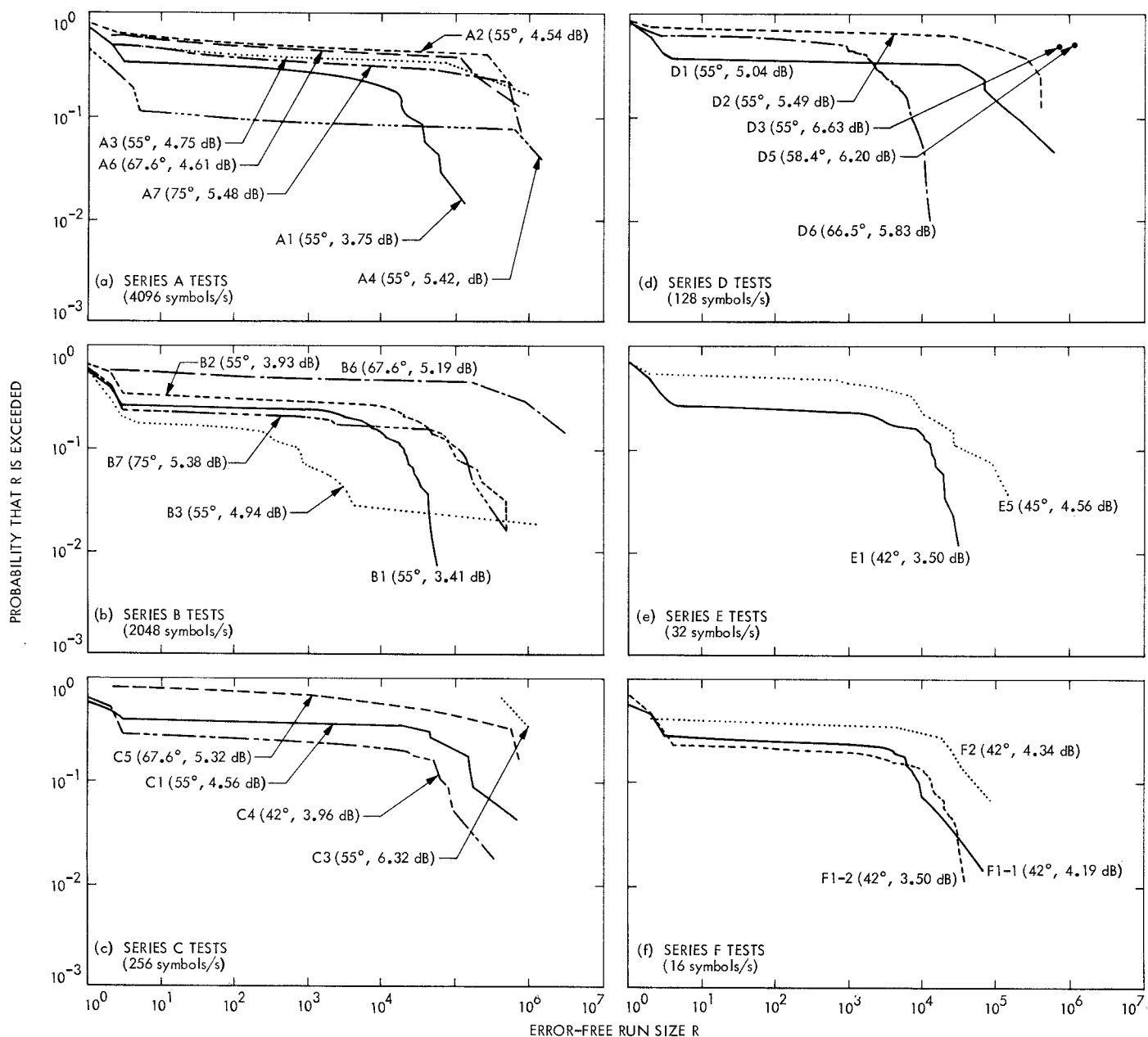


Fig. 5. Error-free run size  $R$  vs probability that  $R$  is exceeded for test series A through F